Relational Algebra

- An Algebra is a Pair: (set of values, set of operations)
- Note that an Algebra is the same idea as an ADT
- Relational Algebra: (relations, relational operators)
  - set of values = relations
  - set of operations = relational operators
- Relational Operators
  - update operators:
    - insert a tuple
    - delete one or more tuples
    - modify one or more tuples
  - retrieval operators: \{σ, π, ∪, \∩, ρ, \times, \mid\mid\}
$\sigma - \text{Selection}$

General Form: $\sigma_{<\text{condition}>} \ <\text{relation}>$

Examples:

$\sigma_{\text{Cost} > 75} \quad \text{RoomNr} \quad \text{Name} \quad \text{NrBeds} \quad \text{Cost}$

<table>
<thead>
<tr>
<th>RoomNr</th>
<th>Name</th>
<th>NrBeds</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kennedy</td>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>Nixon</td>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>Carter</td>
<td>2</td>
<td>80</td>
</tr>
</tbody>
</table>

$\sigma_{\text{ArrivalDate} = 10 \ \text{May} \land \text{NrDays} > 2} \quad \text{S}$

<table>
<thead>
<tr>
<th>GuestNr</th>
<th>RoomNr</th>
<th>ArrivalDate</th>
<th>NrDays</th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>3</td>
<td>10 May</td>
<td>5</td>
</tr>
</tbody>
</table>

RelAlg: 2
π – Projection

General Form: $\pi_{\text{attributes}} \; <\text{relation}>$

Examples:

$$\pi_{\text{City}} \; g$$

$$\pi_{\text{GuestNr}, \text{RoomNr}} \; s$$

<table>
<thead>
<tr>
<th>City</th>
<th>GuestNr</th>
<th>RoomNr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>Hartford</td>
<td>101</td>
<td>2</td>
</tr>
<tr>
<td>Providence</td>
<td>101</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>102</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>103</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>104</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>105</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>106</td>
<td>2</td>
</tr>
</tbody>
</table>

RelAlg: 3
Closed Set of Operators

- Results are relations
- Closed implies we can nest operations in expressions
- Example:

\[ \Pi_{\text{GuestNr}, \text{RoomNr}} \sigma_{\text{ArrivalDate} = 10 \text{ May}} S \]

<table>
<thead>
<tr>
<th>GuestNr</th>
<th>RoomNr</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>102</td>
<td>3</td>
</tr>
<tr>
<td>104</td>
<td>4</td>
</tr>
</tbody>
</table>
Set Operators: $\cap$, $-$, $\cup$

$\pi_{\text{Name}} \ r \cap \pi_{\text{Name}} \ g$

Name
-----
Carter
Green

$\pi_{\text{RoomNr}} \ r - \pi_{\text{RoomNr}} \ S$

RoomNr
-----
5

$\pi_{\text{RoomNr}} \sigma_{\text{Cost} < 75} \ r \cup \pi_{\text{RoomNr}} \sigma_{\text{ArrivalDate} = 10 \text{ May}} \ S$

RoomNr
-----
4
5
1
3

Note: schemes must be compatible.
\( \rho - \text{Renaming} \)

**General Form:**
\( \rho \ <\text{old attribute}> \leftarrow <\text{new attribute}> \ <\text{relation}> \)

**Examples:**

\[
\pi_{\text{RoomName}} \rho_{\text{Name} \leftarrow \text{RoomName}} \sigma_{\text{Cost} < 75} r
\]

\[
\begin{array}{l}
\text{RoomName} \\
\underline{\text{--------------}} \\
\text{Blue} \\
\text{Green}
\end{array}
\]

\[
\pi_{\text{Nr, Name}} \rho_{\text{RoomNr} \leftarrow \text{Nr}} r \cap \pi_{\text{Nr, Name}} \rho_{\text{GuestNr} \leftarrow \text{Nr}} g
\]

\[
\begin{array}{l}
\text{Nr} \quad \text{Name} \\
\underline{\text{--------------}}
\end{array}
\]

**Note:** The old attribute must be in the scheme and the new attribute must not be in the scheme.
\( \times \ - \text{Cross Product} \)

\[ \prod_{\text{NrBeds}} r \times \prod_{\text{RoomNr}} s \]

\[
\begin{array}{ccc}
\text{NrBeds} \times & \text{RoomNr} & = \\
2 & 1 & 2 \\
1 & 2 & 2 \\
1 & 3 & 3 \\
1 & 4 & 4 \\
\end{array}
\]

Note: The intersection of the schemes must be empty.
\[ |x| \text{ - Natural Join} \]

\[
r \times g =
\pi_{\text{RoomNr}, \text{Name}, \text{NrBeds}, \text{Cost}, \text{GuestNr}, \text{StreetNr}, \text{City}}
\sigma_{\text{Name} = \text{Name}'}(r \times \rho_{\text{Name} = \text{Name}'} g)
\]

<table>
<thead>
<tr>
<th>RoomNr</th>
<th>Name</th>
<th>NrBeds</th>
<th>Cost</th>
<th>GuestNr</th>
<th>StreetNr</th>
<th>City</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Carter</td>
<td>2</td>
<td>80</td>
<td>102</td>
<td>10 Main</td>
<td>Hartford</td>
</tr>
<tr>
<td>5</td>
<td>Green</td>
<td>1</td>
<td>50</td>
<td>105</td>
<td>10 Main</td>
<td>Boston</td>
</tr>
</tbody>
</table>

RelAlg: 8
Natural Join – Examples

A B  |   B C   =  A B C
------|-------|------
1 2   |   1 2  =  1 2 3
3 2   |   2 3  3 2 3
4 5   |   5 7  4 5 7
6 7   |   5 8  4 5 8

A B  |   C D   =  A B C D
------|-------|------|------
1 2   |   1 3  =  1 2 1 3
3 4   |   2 4

A B C  |   B C D   =  A B C D
--------|-------|------|------|------
1 2 3  |   2 3 4  =  1 2 3 4
2 2 3  |   5 6 7
4 5 6  |   2 6 0  4 5 6 7

RelAlg: 9
Query Examples

List names and cities of guests arriving on 15 May.

$$\Pi_{\text{Name, City}} \sigma_{\text{ArrivalDate} = 15 \text{ May}} (g \mid \times \mid s)$$

List names of each guest who has a reservation for a room that has the same name as the guest’s name.

$$\Pi_{\text{Name}} (g \mid \times \mid s \mid \times \mid r)$$

List names of guests who have a reservation for rooms with two beds.

$$\Pi_{\text{Name}} (g \mid \times \mid s \mid \times \mid \Pi_{\text{RoomNr}} \sigma_{\text{NrBeds} = 2} r)$$

RelAlg: 10
More Query Examples

List the names of guests from Hartford who are arriving after 10 May.

$$\Pi_{\text{Name}} (\Pi_{\text{GuestNr,Name}} \sigma_{\text{City = Hartford}} g \times \Pi_{\text{GuestNr}} \sigma_{\text{ArrivalDate > 10 May}} s)$$

List names of rooms for which no guest is arriving on 10 May.

$$\Pi_{\text{Name}} (r \times (\Pi_{\text{RoomNr}} r \quad - \Pi_{\text{RoomNr}} \sigma_{\text{ArrivalDate = 10 May}} s))$$